Performance of Full-Spectrum Combining for the Galileo S-Band Mission *

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Abstract

This article describes the performance of Full Spectrum Combining (FSC), an arraying technique being considered for the Galileo S-Band mission, in terms of symbol SNR degradation and symbol SNR loss. It is shown that both degradation and 10ss are in agreement at low values of symbol SNR but diverge at high values. For the Galileo S-band mission, the degradation provides a good estimate for the performance as the symbol SNR is typically below -5 dB. Depending on the subcarrier bandwidth, the degradation for 2 70-meter antennas can vary from 0.1 dB to 0.5 dB.

I Introduction

In deep space communications, combining the outputs of multiple antennas is commonly referred to as allaying. Arraying techniques arc of significant importance because systems that employ arraying have better performance than those that don 't! For example, if signal power-to-noise density ratio (P/N_0) is a measure of system performance, then the effective P/N_0 after arraying is ideally equal to the sum of the P/N_0's corresponding to the individual antennas. Although arraying improves system performance, it has been employed sparingly in the past because arraying adds complexity. Consequently, it has been most appealing as a technique to increase the communications link margin only when links are operating near threshold. For instance consider the Galileo spacecraft which, due to a malfunctional

high gain antenna, must rely on a low gain antenna (and a much reduced link margin) for data transmission to earth. The Galileo S-Band mission will employ arraying (and other techniques such as suppressed carriers and data compression) to improve its link margin and maximize data return. This paper describes the performance of Full-Spectrum Combining (FSC) which is one of the arraying technique being considered for the Galileo S-Band mission. In this scheme, depicted in Figure 1, the received signal at each antenna is down-converted to an intermediate frequency (IF), brought to a central location, and combined before being demodulated by a single receiver consisting of a car ier, a   a symbol synchronization loop, and a matched filter.

In this paper, the performance of FSC is measured both in terms of symbol SNR degradation and symbol SNR loss. Symbol SNR degradation is defined as the ratio of the SNR at the symbol matched filter output in the presence of non-ideal synchronization to the ideal SNR that can be attained, which assumes no combining or synchronization losses. Symbol SNR loss, on the other hand, is defined as the additional symbol SNR needed in the presence of imperfect synchronization to achieve the same symbol error rate (SER) as in the presence of perfect synchronization. Comparatively, the latter gives the absolute while the former gives the relative performance advantage of FSC. Moreover, degradation is less computationally demanding than loss. In the next section we derive the symbol SNR degradation and loss for FSC and, afterwards, conclude with a numerical example.

2   FSC Performance

In FSC, signals at multiple antennas are combined at IF and demodulated by a single receiver, as shown in Figure 1. Assuming an array of L antennas, the ith IF signal at the combiner input is given as [2]

$$r_i(t) = \sqrt{2P_i}d(t - \tau_i)Sin(\omega_{sc}(t - \tau_i) + \theta_{sc})\cos(\omega_I t - \omega_c\tau_i + \theta_c) + n_i(t) \tag{1}$$

for $i = 1,...,L$, where P_i is the received power at antenna i in Watts (W), ω_c and ω_I are the angular frequencies of the received carrier at the antenna and at IF, $Sin(\omega_{sc}t + \theta_{sc})$ denotes the squarewave subcarrier with frequency ω_{sc} and phase θ_{sc}, $d(t)$ the binary ± 1 data with rate $\frac{1}{T}$ and θ_c the carrier phase. The quantity τ_i denotes the time delay in signal reception between the first and ith antennas ($\tau_1 = 0$). Note that delaying the signals by τ_i at IF is not sufficient for combining the signals. An additional phase shift is required to compensate for the $\omega_c\tau_i$ term. The narrow-band noise $n_i(t)$ can be written as

$$n_i(t) = \sqrt{2}n_{ci}(t)\cos(\omega_I t - \theta_I) - \sqrt{2}n_{si}(t)\sin(\omega_I t - \theta_I) \tag{2}$$

where $n_{ci}(t)$ and $n_{si}(t)$ are statistically independent, stationary bandli ited white Gaussian noise processes with one-sided spectral density N_{0i} W/Hz.

2

In order to maximize the combining gain, the IF signals are aligned in time and phase before being combined [2]. In a practical system, time misalignment and phase mismatch results in a less than ideal P/N_0 gain at the output of the combiner. The analysis below assumes that the signals are perfectly aligned in time but not in phase. Denoting the phase alignment error as $\Delta\phi_{i1}$, the combined signal power condition on $\Delta\phi_{i1}$ is given as [2]

$$P'_z = P_1 \sum_{i=1}^{L} \sum_{j=1}^{L} \gamma_i \gamma_j C_{ij} \tag{3}$$

where $\gamma_i = \frac{P_i}{P_1} \frac{N_{01}}{N_{0i}}$ and

$$C_{ij} = e^{j(\Delta\phi_{i1} - \Delta\phi_{j1})} \tag{4}$$

is the complex signal reduction function due to phase misalignment.

After carrier and subcarrier demodulation, the combined symbol stream at the matched filter output can be written as [1]

$$v_k = \begin{cases} \sqrt{P''_d} C_c C_{sc} d_k + n_k & d_k = d_{k-1} \\ \sqrt{P''_d} C_c C_{sc}(1 - \frac{1}{\pi}|\phi_{sy}|) d_k + n_k & d_k \neq d_{k-1} \end{cases} \tag{5}$$

where the combined data power $P''_d = P'_z \sin^2 \Delta$ (Δ is the modulation index), and $d_k = \pm 1$ is the kth symbol. Moreover, the noise n_k is a Gaussian random variable with variance $\sigma_n^2 = \frac{N_{0_{eff}}}{2T}$ where T is the symbol period and $N_{0_{eff}}$ (in W/Hz) is the effective combined one-sided noise spectral density level at the match filter input given by [2]

$$N_{0_{eff}} = N_{01} \sum_{i=1}^{L} \gamma_i^2 \tag{6}$$

The signal reduction functions $C_c = \cos\phi_c$ and $C_{sc} = 1 - \frac{2}{\pi}|\phi_{sc}|$ in (5) are respectively due to imperfect carrier and subcarrier synchronization. The quantities ϕ_c and ϕ_{sc} respectively denote the carrier and subcarrier phase tracking errors. The signal reduction due to symbol timing error, which occurs only during symbol transitions, is equal to $1 - \frac{1}{\pi}|\phi_{sy}|$ where ϕ_{sy} denotes the symbol phase tracking error. The SNR conditioned on $\Delta\phi_{i1}, \phi_c, \phi_{sc}, \phi_{sy}$, denoted SNR', is defined as the square of the conditional mean of v_k divided by the conditional variance of v_k, i.e,

$$SNR' = \begin{cases} \frac{2P''_d T}{N_{0_{eff}}} C_c^2 C_{sc}^2 & d_k = d_{k-1} \\ \frac{2P''_d T}{N_{0_{eff}}} C_c^2 C_{sc}^2 (1 - \frac{1}{\pi}|\phi_{sy}|)^2 & d_k \neq d_{k-1} \end{cases} \tag{7}$$

The last equation is useful in computing the symbol SNR degradation and loss as shown below.

2.1 Symbol SNR Degradation

The SNR degradation is defined as the ratio of the SNR in the presence of imperfect phase alignment and synchronization to ideal SNR (no phase errors). After computing the SNR in the presence of phase errors (obtained by averaging (7) over $\Delta\phi_{i1}, \phi_c, \phi_{sc},$ and ϕ_{sy}) and then dividing by the ideal SNR given by [2] (i.e., $SNR_{ideal} = \frac{2P_{d1}T}{N_{01}}\sum_{i=1}^{L} \gamma_i$), yields the SNR degradation, namely,

$$D = -10\log_{10}\left(\overline{C_c^2}\, \overline{C_{sc}^2}\, \overline{C_{sy}^2}\left(\frac{\sum_{i=1}^{L}\gamma_i^2 + \sum_{n=1}^{L}\sum_{\substack{m=1\\n\neq m}}^{L}\gamma_n\gamma_m\overline{C_{nm}}}{\sum_{n=1}^{L}\gamma_n}\right)\right) \tag{8}$$

where

$$\overline{C_c^2} = \frac{1}{2}\left[1 + \frac{I_2(\rho_c)}{I_0(\rho_c)}\right] \tag{9}$$

$$\overline{C_{sc}^2} = 1 - \sqrt{\frac{8}{\pi^2}\cdot\frac{1}{\sqrt{\rho_{sc}}}} + \frac{4}{\pi^2}\cdot\frac{1}{\rho_{sc}} \tag{10}$$

$$\overline{C_{sy}^2} = 1 - \sqrt{\frac{1}{\pi^2}\cdot\frac{1}{\sqrt{\rho_{sy}}}} + 1 - \frac{1}{4\pi^2}\cdot\frac{1}{\rho_{sy}} \tag{11}$$

where $I_k(\rho_c)$ denotes the modified Bessel function of order k. In deriving; (9)-(11), ϕ_c was assumed to be Tikhonov distributed with $\sigma_c^2 = \frac{1}{\rho_c}$, and ϕ_{sc} and ϕ_{sy} were assumed to be Gaussian with respective $\sigma_{sc}^2 = \frac{1}{\rho_{sc}}$ and $\sigma_{sy}^2 = \frac{1}{\rho_{sy}}$ where the loop SNRs $\rho_c, \rho_{sc},$ and ρ_{sy} are given as [2]

$$\rho_c = \frac{P_d T}{N_{0_{eff}}B_c}\left(1 + \frac{1}{2P_d T/N_{0_{eff}}}\right)^{-1} \tag{12}$$

$$\rho_{sc} = \left(\frac{\pi}{2}\right)^2\frac{1}{N_0 W_{sc}B_{sc}}\left(1 + \frac{1}{2P_d T/N_{0_{eff}}}\right)^{-1} \tag{13}$$

$$\rho_{sy} = \frac{1}{2\pi^2}\frac{1}{N_0 W_{sy}B_{sy}}\,\mathrm{erf}^2\left(\sqrt{P_d T/N_{0_{eff}}}\right) \tag{14}$$

Where P_d is the average combined data power found by averaging the conditional data power in (5) over the residual phases. The quantities B_c, B_{SC}, and B_{sy} Hz denote the single-sided carrier, subcarrier, and symbol loop) bandwidth), respectively. The parameters W_{sc} and W_{sy}, which denote the subcarrier and symbol window, are unitless and limited to (0, 1]. Referring to (8), the average signal reduction function due to phase misalignment between IF signals n and m, denoted $\overline{C_{nm}}$, is given as [2]

$$\overline{C_{nm}} = \begin{cases} e^{-\frac{1}{2}[\sigma_{\Delta\phi_{n1}}^2 + \sigma_{\Delta\phi_{m1}}^2]} & n \neq m \\ 1 & n = m \end{cases} \tag{15}$$

4

where $\sigma^2_{\Delta\phi_{n1}}$ is the variance of $\Delta\phi_{n1}$ which was assumed to be Gaussian with zero mean and variance

$$\sigma^2_{\Delta\phi_{n1}} = \frac{1}{2SNR_{n1}} \tag{16}$$

where SNR_{n1} is the SNR of the correlator in Fig. 1 and is given as [2]

$$SNR_{n1} = \frac{P_{d1}}{N_{01}} \frac{2T_{corr}}{[1 + 1/\gamma_i + (B_{corr}N_{0n}/P_n)]} \tag{17}$$

with B_{corr} denoting the single-sided bandwidth of the IF filter preceding the correlator and T_{corr} the averaging time of the correlator.

2.2 Symbol SNR Loss

The FSC SER for an L antenna array, denoted $P_s(E)$, is defined as

$$P_s(E) = \underbrace{\int_{-\pi}^{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}}_{L-1} P'_s(E)\left[p(\phi_c)p(\phi_{sc})p(\phi_{sy})(\prod_{n=2}^{L}p(\Delta\phi_{n1}))\right]d\Delta\phi d\phi_{sy}d\phi_{sc}d\phi_c \tag{18}$$

where $\Delta\phi = (\Delta\phi_{21},\ldots,\Delta\phi_{(L-1)1})$, the $L-1$ residual phase errors between the reference signal and the $L-1$ remaining signals are independent Gaussian random variables with variance given by (16). The phases ϕ_c, ϕ_{sc}, and ϕ_{sy} are statistically described as in (8). Following similar mathematical manipulation as in [3], the conditional SER, becomes

$$P'_s(E) = \frac{1}{4}\text{erfc}[\sqrt{SNR'}\text{when } d_k \neq d_{k-1}] + \frac{1}{4}\text{erfc}[\sqrt{SNR'}\text{when } d_k = d_{k-1}] \tag{19}$$

Substituting (7) for SNR' yields

$$P'_s(E) = \frac{1}{4}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}\frac{(\sum_{n=1}^{L}\gamma_n^2 + \sum_{n=1}^{L}\sum_{\substack{m=1\\n\neq m}}^{L}\gamma_n\gamma_mC_{nm})}{(\sum_{n=1}^{L}\gamma_n)}C_cC_{sc}(1 - \frac{1}{2\pi}|\phi_{sy}|)}\right]$$

$$+ \frac{1}{4}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}\frac{(\sum_{n=1}^{L}\gamma_n^2 + \sum_{n=1}^{L}\sum_{\substack{m=1\\n\neq m}}^{L}\gamma_n\gamma_mC_{nm})}{(\sum_{n=1}^{L}\gamma_n)}C_cC_{sc}}\right] \tag{20}$$

Where $E_s/No = P_{d1}T'/N_{0l}$ is the symbol SNR at antenna 1 and the complementary error function is defined as

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_{x}^{\infty}\exp(-v^2)dv \tag{21}$$

5

3 Galileo S-band Mission Scenario

The FSC symbol SNR degradation and symbol SNR loss for the case of two 70-1 neter antennas is depicted in Figures 2 and 3. In particular, Figure 2 was computed using typical values for the Galileo S-Band mission: $\frac{P_1}{N_{0_1}}$: $\frac{P_2}{N_{0_2}}$ 15 dB-Hz, R_s = 400 syII1/see, B_c = 0.1 Hz, B_{corr} = 10,000 Hz, and T_{corr} = 120 sec, which corresponds to the ideal SER of 0.287169. It is evident from the Figure 2 that for this SER, symbol SNR degradation and loss agree to within 0.01 dB.

The symbol SNR degradation in Figure 2 was found by using (8), where the earlier, subcarrier, symbol, and correlation loop SNR's were derived using (12), (1 3), (1 4), and (17) respectively; these loop SNR's are summarized in Table 1a. For the correlation loop SNR, we assume that the correlation bandwidth allows only the first harmonic of the subcarrier squarewave to pass unfiltered, resulting in 0.9 dB loss in power.

The symbol SNR loss, on the other hand, was obtained by using (18) for the case of two 70-meter antennas and, consequently, the conditional SER given in (20) simplifies to

$$P_s''(E) = \frac{1}{4}\mathrm{erfc}\left[\sqrt{\frac{E_s}{N_0}}(1+\cos\Delta\phi_{21})C_cC_{sc}(1-\frac{1}{\pi}|\phi_{sy}|)\right] + \frac{1}{4}\mathrm{erfc}\left[\sqrt{\frac{E_s}{N_0}}(1+\cos\Delta\phi_{21})C_cC_{sc}\right]$$

(22)

The carrier, subcarrier, symbol, and correlation loop SNR's are summarized in Table 1a, with E_s/N_0 = -11 dB. The symbol SNR can now be found by iteratively solving for the additional SNR needed to achieve the ideal SER of 0.287169. The process of finding the symbol SNR loss is illustrated for the case when the subcarrier and symbol loop bandwidths are equal to 1 mHz. Using E_s/N_0 of -11 dB, the SER using (1 8) becomes 0.201 074. Since this is higher that the ideal SER, E_s/N_0 is increased by some $\Delta E_s/N_0$ so that the SER using (18) is equal to the ideal SER. After an iterative process, $\Delta E_s/N_0$ of 0.18 dB achieves the ideal SER which by definition is the symbol SNR loss. The same iterative procedure was used to generate the symbol SNR loss for different subcarrier and symbol loop bandwidths as shown in Figure 2.

The performance of FSC for an ideal SER of 3.4 x 10^{-5} is presented in Figure 3 which has the same values as in Figure 2 except $\frac{P_1}{N_{01}}$: $\frac{P_2}{N_{Q2}}$: 32 dB-Hz and B_c = 250 Hz. Both the symbol SNR degradation and loss were computed using the same procedure as before wit,]] E_s/N_0 in this case equal to 6 dB, and the loop SNR's of the carrier, subcarrier, symbol, and correlator are summarized in Table 2b. From Figure 3, it is evident that symbol SNR loss is about 1.4 dB larger than symbol SNR degradation. This is to be expected since the SER for this case is high. In general, symbol SNR loss gives the absolute performance advantage of an arraying scheme while symbol SNR degradation gives the relative performance advantage. For low symbol SNRs, however, degradation and loss are comparable as shown in Figure 2. Likewise, symbol SNR degradation is a lower bound for symbol SNR loss. Computationally,

s-y mbol SNR degradation is easier to calculate than symbol SNR 10SS; the latter requires numerically integrating the SER since no exact dosed form solution exists and for large L consumes a lot of computer time.

4 Conclusion

This paper described the performance of FSC using symbol SNR degradation and 10SS. It is shown that both degradation and loss are in agreement at low values of symbol SNR but diverge at nigh values. For the Galileo S-band mission, the degradation provides a good estimate for the performance as the symbol SNR is typically below -5 dB. Depending on the subcarrier bandwidth, the degradation for 2 70-meter antennas can vary from 0.1 dB to 0.5 dB.

Acknowledgments

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